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Superconformal mechanics and the super Virasoro algebra

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Abstract: We consider $\mathcal{N}=1,2$ superconformal mechanics in $0+1$ dimensions and show that if the hamiltonian is invertible the superconformal generators can be used to construct half of the super Virasoro algebra. The full algebra can be derived if the special conformal generator is also invertible. The generators are quantized and a general prescription is given for the construction of the $\mathcal{N}=1$ algebra independently of the specific details of the superconformal mechanics provided that in addition its quadratic Casimir operator vanishes.

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Contents

1. Introduction

There is an ongoing interest in conformal mechanics, since the early work of [1], and in the supersymmetric generalization [2, 3, 4]. These one dimensional conformal field theories admit exact solutions to problems that can be accessed only perturbatively in higher dimensions, due to the existence of the powerful conformal symmetry that constrains the dynamics. Although these $d = 1$ conformal mechanics are relatively simple they are still not trivial. A geometrical picture that relates the one dimensional field equations to geodesics in the group space of $\mathrm{SO}(1,2)$ and $\mathrm{SU}(1,1|1)/\mathrm{U}(1)$ was constructed in [5, 6].

Recently, the AdS_{p+2}/CFT_{p+1} conjecture, see e.g. [7, 8], has raised a new interest in conformal mechanics, i.e. the case of $p = 0$. This conjecture proposes that in an appropriate limit certain conformal field theories in $p+1$ dimensions are dual to superstring theory on an AdS space in $p+2$ dimensions times some compact manifold. The case of conformal quantum mechanics and the corresponding AdS_2 [9] may offer more insights to this conjecture due to the simplification of an analysis in so few dimensions, though there are also new features since the boundary of the AdS_2 is not connected.

Although the basis to this paper is mainly algebraic, it is instructive to bear in mind concrete realizations of physical systems which are governed by (super) conformal mechanics. An illustrative example is that of the physics of a test particle in the near horizon region of a $d = 4$ $\mathcal{N} = 2$ extreme Reissner-Nordström black hole [10, 11]. This particle is described by conformal mechanics where the canonical coordinates are the radial coordinate and radial momentum in $d = 4$. The main interest in such systems is due to the fact that black holes provide the arena where gravity and quantum mechanics match [12]. Understanding these systems shed light on the intriguing problems of quantum gravity. Another example is that of a non-relativistic spinning particle coupled to a magnetic field and a scalar potential [13].[1]

In this paper we follow the analysis in [14, 15] on conformal symmetry and the Virasoro algebra and extend it to the case of $\mathcal{N} = 1, 2$ super conformal symmetry. In section 2 we identify the superconformal algebra as the subalgebra of the super Virasoro algebra. In section 3 we obtain classical recursion equations for the generators and find a representation of the full algebra for a free particle and an interacting one. At the level of Poisson brackets this algebra close to $\mathcal{N} = 1, 2$ Neveu-Schwarz super Virasoro algebra, with a U(1) Kac-Moody algebra [16] for the latter case. In section 4 we describe the quantization of the generators and give a general construction of $\mathcal{N} = 1$ super Virasoro generators out of the superconformal ones.

2. Generators of superconformal quantum mechanics

The generators of $0+1$ dimensional superconformal quantum mechanics obey the super algebra of $\mathrm{osp}(2|2) \cong \mathrm{su}(1,1|1)$ [11]. Their non-trivial commutation and anti-commutation relations can be written in the following way[2]

$$
\begin{aligned}
&[H, D]_- = iH\,, && [K, D]_- = -iK\,, && [H, K]_- = 2iD\,, \\
&[Q_i, Q_j]_+ = 2\delta_{ij} H\,, && [S_i, S_j]_+ = 2\delta_{ij} K\,, && [Q_i, S_j]_+ = 2\delta_{ij} D + \epsilon_{ij} B\,, \\
&[D, Q_i]_- = -\frac{i}{2} Q_i\,, && [D, S_i]_- = \frac{i}{2} S_i\,, && \\
&[K, Q_i]_- = -iS_i\,, && [H, S_i]_- = iQ_i\,, && \\
&[B, Q_i]_- = -i\epsilon_{ij} Q_j\,, && [B, S_i]_- = -i\epsilon_{ij} S_j\,, && i, j = 1, 2\,,
\end{aligned}
\tag{2.1}
$$

where $[\,,\,]_\pm$ stands for commutators and anticommutators. The following identification of the generators

$$
\begin{aligned}
&L_{-1} = H\,, && L_0 = -D\,, && L_1 = K\,, \\
&G^i_{-1/2} = Q_i\,, && G^i_{1/2} = -S_i\,, && B_0 = B
\end{aligned}
\tag{2.2}
$$

[1]For this system to be (super) conformal invariant one need to make diffeomorphisms transformations on the background to compensate on the conformal transformations.

[2]We rescaled the fermionic charges by a factor $\sqrt{2}$ relative to [11].

can be used to recast the algebra (2.1) as a subalgebra of the super Virasoro algebra with a U(1) charge

$$\begin{aligned}
[L_m, L_n]_- &= i(m-n)L_{m+n} \\
[L_m, G^i_r]_- &= i\left(\frac{1}{2}m - r\right) G^i_{m+r} \\
[G^i_r, G^j_s]_+ &= 2\delta_{ij} L_{r+s} + \epsilon_{ij}(r-s)B_{r+s} \\
[B_k, L_m]_- &= ikB_{k+m} \\
[B_k, G^i_r]_- &= -i\epsilon^{ij} G^j_{k+r}
\end{aligned} \tag{2.3}$$

for $m, n = -1, 0, 1$, $r, s = -1/2, 1/2$ and $k = 0$. A realization of this algebra is given for the theory of a free particle by

$$\begin{aligned}
K &= \frac{1}{2}x^2\,, \qquad & D &= -\frac{1}{4}(xp + px)\,, \qquad & H &= \frac{1}{2}p^2 \\
Q^i &= \psi^i p\,, \qquad & S^i &= -\psi^i x\,, \qquad & B &= \frac{i}{2}[\psi^1, \psi^2]_-
\end{aligned} \tag{2.4}$$

where ψ^i, $i = 1, 2$ are Grassmann coordinates $[\psi^i, \psi^j]_+ = \delta^{ij}$.

For completion we give an explicit form of the conserved charges associated with this algebra written in a way that is compatible with the whole super Virasoro algebra. Time translation is generated by $H = L_{-1}$ so for any quantum generators $\mathcal{G}$ the Heisenberg representation gives

$$\mathcal{G}(t) = e^{iL_{-1}t}\mathcal{G}e^{-iL_{-1}t} = \sum_{l=0}^{\infty} \frac{(-it)^l}{l!}[[\mathcal{G}, L_{-1}], L_{-1}], \dots, L_{-1}]\,, \tag{2.5}$$

where the first term corresponding to $l = 0$ is $\mathcal{G}$. The nested commutators are easily calculated for any super Virasoro generator

$$\begin{aligned}
G^i_r(t) &= \sum_{l=0}^{\infty} \binom{r+1/2}{l} G^i_{r-l} t^l \\
L_n(t) &= \sum_{l=0}^{\infty} \binom{n+1}{l} L_{n-l} t^l \\
B_k(t) &= \sum_{l=0}^{\infty} \binom{k}{l} B_{k-l} t^l\,,
\end{aligned} \tag{2.6}$$

which are finite sums for $r \geq -1/2$, $n \geq -1$ and $k \geq 0$. If, for example, $k = -m < 0$ we may use analytic continuation

$$l!\binom{k}{l} \to \lim_{\epsilon \to 0} \frac{\Gamma(-m+1+\epsilon)}{\Gamma(-m-l+1+\epsilon)} = (-1)^l \frac{\Gamma(m+l)}{\Gamma(m)}\,.$$

The generators in (2.6) are conserved by construction

$$\frac{d\mathcal{G}}{dt} = i[\mathcal{G}, L_{-1}] + \frac{\partial \mathcal{G}}{\partial t} = 0\,. \tag{2.7}$$

3. Classical algebra

In order to find the complete super Virasoro algebra we use the classical algebra

$$\begin{aligned}
\{L_m, L_n\} &= (m-n)L_{m+n} \\
\{L_m, G_r^i\} &= \left(\frac{1}{2}m - r\right) G_{m+r}^i \\
\{G_r^i, G_s^j\} &= 2\delta_{ij}L_{r+s} + \epsilon_{ij}(r-s)B_{r+s} \\
\{B_k, L_m\} &= kB_{k+m} \\
\{B_k, G_r^i\} &= -\epsilon^{ij}G_{k+r}^j \\
\{B_k, B_m\} &= 0\,,
\end{aligned} \tag{3.1}$$

where $\{\ ,\ \}$ denote the even Poisson brackets [17]

$$\{G, K\} = \frac{\partial G}{\partial x}\frac{\partial K}{\partial p} - \frac{\partial G}{\partial p}\frac{\partial K}{\partial x} - \sum_{l=1}^{2}(-1)^{p(G)}\frac{\partial G}{\partial \theta^l}\frac{\partial K}{\partial \theta^l}\,, \tag{3.2}$$

with $p(Q) = 1$ for an odd generator and zero otherwise. This definition gives for even coordinates $xp - px = 0$

$$\{x, p\} = 1 \tag{3.3}$$

and for odd coordinates $\theta^i\theta^j + \theta^j\theta^i = 0$

$$\{\theta^i, \theta^j\} = \delta^{ij}\,. \tag{3.4}$$

In what will follow we use the explicit representation of the global generators, e.g. the generators that are given in equation (2.4), and the classical algebra to obtain a set of solvable differential equations. This is done by calculating the Poisson brackets of a general generator of the algebra with one of the global generators and demanding that it will be equal to the r.h.s. of the algebra (3.1), as was done in [14] for the L_n generators.

3.1 Free particle

We begin with the super conformal generators of the free particle that are given in equation (2.4). The Poisson brackets of the G_r^i generators with the hamiltonian and the special conformal generator gives

$$\begin{aligned}
-p\frac{\partial G_r^i}{\partial x} &= \{H, G_r^i\} = \{L_{-1}, G_r^i\} = \left(-\frac{1}{2} - r\right) G_{r-1}^i \\
x\frac{\partial G_r^i}{\partial p} &= \{K, G_r^i\} = \{L_1, G_r^i\} = \left(\frac{1}{2} - r\right) G_{r+1}^i\,.
\end{aligned} \tag{3.5}$$

The integrability condition $\partial^2 G^i_r/\partial p \partial x = \partial^2 G^i_r/\partial x \partial p$ gives the recursion formula for G^i_r

$$\frac{1}{x^2}\left(r-\frac{1}{2}\right)G^i_{r+1} = -\frac{1}{p^2}\left(r+\frac{1}{2}\right)G^i_{r-1} + \frac{2r}{xp}G^i_r\,. \tag{3.6}$$

Similar equations for L_n and B_k exist and are easily solved. The classical Virasoro generators are then given by

$$\begin{aligned} G^i_r &= \theta^i x^{1/2+r} p^{1/2-r} \\ L_n &= \frac{1}{2} x^{1+n} p^{1-n} \\ B_k &= B\, x^k p^{-k}\,, \end{aligned} \tag{3.7}$$

for $n, k \in \mathbb{Z}$ and $r \in \mathbb{Z}+1/2$, $B = \frac{1}{2}[\theta^1, \theta^2]$. For $n \geq -1$, $r \geq -1/2$ and $k \geq 0$ we can recast these generators as

$$\begin{aligned} L_n &= L_{-1}(L_0 L_{-1}^{-1})^{n+1} \\ G^i_r &= G^i_{-1/2}(L_0 L_{-1}^{-1})^{r+1/2} \\ B_k &= B_0 (L_0 L_{-1}^{-1})^k\,. \end{aligned} \tag{3.8}$$

This is an extension to [15]. The representation, however, is not "unitary" $L_n^* \neq L_{-n}$. To get a unitary representation we consider the linear combination of DFF [1]

$$\begin{aligned} L_1 &= \frac{1}{2}\left(aH - \frac{K}{a} - 2iD\right) = \frac{1}{2}z^2 \\ L_0 &= \frac{1}{2}\left(aH + \frac{K}{a}\right) = \frac{1}{2}zz^* \\ L_{-1} &= \frac{1}{2}\left(aH - \frac{K}{a} + 2iD\right) = \frac{1}{2}z^{*2}\,, \end{aligned} \tag{3.9}$$

where we define

$$z = \frac{\sqrt{a}p - i\frac{x}{\sqrt{a}}}{\sqrt{2}}\,, \qquad z^* = \frac{\sqrt{a}p + i\frac{x}{\sqrt{a}}}{\sqrt{2}}\,. \tag{3.10}$$

To obtain the entire algebra we can solve the differential equations again or observe that the new generators have the same form as in equation (3.7) with $x \to z$ and $p \to z^*$. Therefore the solution

$$\begin{aligned} L_n &= \frac{1}{2} z^{1+n} z^{*\,1-n} \\ B_k &= B z^k z^{*\,-k} \\ G^i_r &= \theta^i z^{1/2+r} z^{*\,1/2-r} \end{aligned} \tag{3.11}$$

obey the algebra up to new factors of i due to the fact that the coordinate transformation (3.10) is canonical up to a scale, i.e. $\{z, z^*\} = -i$. Another difference between the two representations, besides unitarity, is that for the generators in (3.11) time translation is generated by L_0 and not by L_{-1}.

3.2 An interacting particle

To obtain the generators of an interacting particle we note that the classical Virasoro algebra is invariant under $L_n \to f^n L_n$, $G^i_r \to f^r G^i_r$ and $B_k \to f^k B_k$ where $f = f(L_0)$. Since for (3.7) $L_0 = 2xp$ we get

$$\begin{aligned} G^i_r &= \theta^i x^{1/2+r} p^{1/2-r} f^r \\ L_n &= \frac{1}{2} x^{1+n} p^{1-n} f^n \\ B_k &= B\, x^k p^{-k} f^k \,. \end{aligned} \tag{3.12}$$

The hamiltonian of a particle in a conformal potential is obtained for $f^{-1} = 1 + g/x^2p^2$

$$L_{-1} = H = \frac{1}{2}\left(p^2 + \frac{g}{x^2}\right). \tag{3.13}$$

For a superconformal potential we first recall the general method [18] for supersymmetrizing a classical and quantum mechanical hamiltonian. Given a hamiltonian $H = \frac{1}{2}p^2 + V$ we can define the odd generators

$$Q^i = (p\mathbf{1} + W_{,x}\,\varepsilon)^{ij}\psi^j\,, \tag{3.14}$$

where $W_{,x} = dW/dx$, which obey the following relation

$$\{Q^i, Q^j\} = \delta^{ij}\left(p^2 + W_{,x}^2 - 2B\,W_{,xx}\right). \tag{3.15}$$

By solving $\sqrt{2V} = W_{,x}$ we get

$$\begin{aligned} \{Q^i, Q^j\} &= 2\delta^{ij} H_{\text{susy}} \\ H_{\text{susy}} &= \frac{1}{2}\left(p^2 + {W_{,x}}^2 - 2B\,W_{,xx}\right). \end{aligned} \tag{3.16}$$

The potential for conformal mechanics is $V = g^2/2x^2$, which is solved for $W = \frac{1}{2} g \log x^2$. These expressions for the hamiltonian and the supersymmetric generators motivate the following ansatz

$$\begin{aligned} G^i_r &= x^{1/2+r} p^{1/2-r} f^r \left(1 + \frac{g^2}{u^2}\right)^{-r-1/2} \left(\mathbf{1} + \frac{g}{u}\varepsilon\right)^{ij} \psi^j \\ L_n &= \frac{1}{2} x^{1+n} p^{1-n} f^n \left(1 + \frac{g^2 + 2gB}{u^2}\right)^{-n} \\ B_k &= x^k p^{-k} f^k \left(1 + \frac{g^2}{u^2}\right)^{-k} B\,, \end{aligned} \tag{3.17}$$

which obey (3.1) and can be recast in the form of (3.8).

One can use the classical analog of (2.5) to obtain an explicit representation of the conserved charges associated with the classical generators, e.g. the use of the generators in equation (3.12) will give

$$\begin{aligned}
G^i_r(t) &= \frac{\theta^i p}{\sqrt{f}}\left(\frac{x}{p}f + t\right)^{r+1/2} \\
L_n(t) &= \frac{1}{2f}p^2\left(\frac{x}{p}f + t\right)^{n+1} \\
B_k(t) &= B\left(\frac{x}{p}f + t\right)^{k}\,,
\end{aligned} \tag{3.18}$$

which is the subalgebra of w_∞ obtained in [19].

4. Quantization of the generators

For the quantization of the generators we first rescale the algebra and absorb all factors of i. For clearness we give the $\mathcal{N} = 0, 1, 2$ algebras:

$$\mathcal{N} = 0: \ [L_m, L_n]_- = (m-n)L_{m+n} \tag{4.1}$$

$$\begin{aligned}
\mathcal{N} = 1: \ [L_m, L_n]_- &= (m-n)L_{m+n} \\
[L_m, G_r]_- &= \left(\frac{1}{2}m - r\right)G_{m+r} \\
[G_r, G_s]_+ &= 2L_{r+s}
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
\mathcal{N} = 2: \ [L_m, L_n]_- &= (m-n)L_{m+n} \\
[L_m, G^i_r]_- &= \left(\frac{1}{2}m - r\right)G^i_{m+r} \\
[G^i_r, G^j_s]_+ &= 2\delta^{ij}L_{r+s} + (r-s)\epsilon^{ij}B_{r+s} \\
[B_k, B_n]_- &= 0 \\
[B_k, L_n]_- &= kB_{k+n} \\
[B_k, G^i_r]_- &= -\epsilon^{ij}G^j_{k+r}
\end{aligned} \tag{4.3}$$

and also the Casimir operator of each of the (super) sl$(2,\mathbb{R})$ subalgebras:

$$\begin{aligned}
\mathcal{N} = 0: \quad L^2 &= L_0^2 + L_0 - L_1L_{-1} \\
\mathcal{N} = 1: \ {}_1C^2 &= L^2 - \frac{1}{2}L_0 + \frac{1}{2}G_{1/2}G_{-1/2} \\
\mathcal{N} = 2: \ {}_2C^2 &= L^2 - L_0 + \frac{1}{2}G^i_{1/2}G^i_{-1/2} + \frac{1}{4}B^2
\end{aligned} \tag{4.4}$$

In [14] the quantization of the bosonic classical generators was obtained by a canonical coordinate transformation:

$$q = \frac{p}{2x}\,, \qquad y = x^2\,, \tag{4.5}$$

and setting $f(u) = xp$. The same procedure applied to the $\mathcal{N} = 2$ generators (3.12) gives

$$\begin{aligned} G^i_r &= \psi^i y^{1/2+r} q^{1/2} \\ L_n &= y^{1+n} q \\ B_k &= y^k B \,. \end{aligned} \tag{4.6}$$

There are two problems we have to address before quantization. The first is how to quantize $\sqrt{q}$ and the second is how to order the operators. For the square root of the momentum we define the covariant derivative $\mathcal{D}^i = \partial/\partial\theta^i + \theta^i q$ which obey $\{D^i, D^j\} = 2\delta^{ij} q$ and since the generators are linear in momentum, ordering $y^{n+1}q$ might add a term proportional to y^n. This motivate the following quantum ansatz

$$\begin{aligned} \sqrt{i} G^i_r &= y^{r+1/2}\mathcal{D}^i - i\left(r + \frac{1}{2}\right) y^{r-1/2}\theta^i T \\ iL_n &= y^{1+n} q - i\frac{(n+1)}{2} y^n T \\ iB_k &= y^k B \,, \end{aligned} \tag{4.7}$$

which satisfied the $\mathcal{N} = 2$ super Virasoro algebra for any $n, k \in \mathbb{Z}$ and $r \in \mathbb{Z} + 1/2$ provided

$$\begin{aligned} T &= \theta^i \frac{\partial}{\partial\theta^i} \\ B &= \theta^1\partial_2 - \theta^2\partial_1 \end{aligned} \tag{4.8}$$

and which can be truncated to the $\mathcal{N} = 1$ algebra generators:

$$\begin{aligned} \sqrt{i} G_r &= y^{r+1/2}\mathcal{D} \\ iL_n &= y^{1+n} q - i\frac{(n+1)}{2} y^n T \,, \end{aligned} \tag{4.9}$$

where $T = \theta\partial/\partial\theta$ and $\mathcal{D} = \partial/\partial\theta + \theta\, q$.

In [15] a general construction of two half-Virasoro algebras was given provided L_1 and L_{-1} are invertible. The conditions for combining them into a single Virasoro algebra were

$$L_1 = L_0 L_{-1}^{-1} L_0 \,, \qquad L_{-1} = L_0 L_1^{-1} L_0 \,. \tag{4.10}$$

These conditions are equivalent, when $L_{\pm 1}$ are invertible, to the vanishing of the quadratic Casimir of the $\mathrm{sl}(2,\mathbb{R})$ algebra

$$L^2 = L_0^2 + L_0 - L_1 L_{-1} = 0 \,. \tag{4.11}$$

Moreover if we consider the whole tower of $\mathrm{sl}(2,\mathbb{R})$ subalgebras contained in the Virasoro algebra, i.e. $\{\frac{1}{n}L_n, \frac{1}{n}L_0, \frac{1}{n}L_{-n}\}$ for $n > 0$ with the solution found in [15]

$$\begin{aligned} L_n &= (L_0 L_{-1}^{-1})^n L_0 \\ L_{-n} &= (L_0 L_1^{-1})^n L_0 \end{aligned} \tag{4.12}$$

we can easily verify that all the quadratic Casimir vanish

$$L^2(n) = \frac{1}{n^2}L_0^2 + \frac{1}{n}L_0 - \frac{1}{n^2}L_nL_{-n} = 0 \tag{4.13}$$

We would like to find a similar construction for a representation of the super Virasoro generators which is independent of the specific super conformal system for the $\mathcal{N} = 1$ case (4.2). For this we first note that the quadratic Casimir can be written as

$$\begin{aligned} {}_1C^2 &= L_0^2 + \frac{1}{2}L_0 - L_1L_{-1} + \frac{1}{2}G_{1/2}G_{-1/2} \\ &\equiv \left[G_{1/2}G_{-1/2} - \left(L_0 + \frac{1}{4}\right)\right]^2 - \frac{1}{16} \end{aligned} \tag{4.14}$$

and that the realization of the generators in (4.9) can be thought of as a change of variables which we can invert to obtain

$$\begin{aligned} q &= iL_{-1} \\ T &= 2(G_{1/2}G_{-1/2} - L_0) \\ y &= G_{1/2}G_{-1/2}L_{-1}^{-1} \\ D &= \sqrt{i}G_{-1/2} \\ \theta &= \frac{2}{\sqrt{i}}(G_{1/2}G_{-1/2} - L_0)G_{-1/2}^{-1} \\ \frac{\partial}{\partial\theta} &= -2\sqrt{i}G_{-1/2}(G_{1/2}G_{-1/2} - L_0)\,, \end{aligned} \tag{4.15}$$

with ${}_1C^2 = 0 \Leftrightarrow \theta^2 = 0$. This inversion allows us to rewrite the generators of half the $\mathcal{N} = 1$ Virasoro algebra as functions of only the (super) sl$(2,\mathbb{R})$ subalgebra generators:

$$\begin{aligned} L_n &= \left(G_{1/2}G_{-1/2}L_{-1}^{-1}\right)^{n+1}L_{-1} - (n+1)\left(G_{1/2}G_{-1/2}L_{-1}^{-1}\right)^n(G_{1/2}G_{-1/2} - L_0) \\ G_r &= \left(G_{1/2}G_{-1/2}L_{-1}^{-1}\right)^{r+1/2}G_{-1/2} \end{aligned} \tag{4.16}$$

where $n \geq -1$. If L_1 is also invertible then this solution is valid for $n < -1$ and obeys the full algebra. The solution reduces to (4.12) for $G_{1/2}G_{-1/2} \sim L_0$. In a similar way to the $\mathcal{N} = 0$ case, we can compute the quadratic Casimir for all the (super) sl$(2,\mathbb{R})$ subalgebras generators

$$\begin{aligned} &\frac{1}{2k+1}L_{2k+1}\,, &&\frac{1}{2k+1}L_0\,, &&\frac{1}{2k+1}L_{-2k-1} \\ &\frac{1}{\sqrt{2k+1}}G_{k+1/2}\,, &&\frac{1}{\sqrt{2k+1}}G_{-k-1/2} \end{aligned} \tag{4.17}$$

The quadratic Casimir of the subalgebras are fix, i.e. $L^2(k) = L^2$ and ${}_1C_2(k) = {}_1C_2 = 0$.

5. Conclusions

We construct the $\mathcal{N} = 1, 2$ super Virasoro algebra out of the superconformal generators at the classical level of Poisson brackets. The generators are ordered and quantized. These quantum generators define new coordinates that are inverted and used to construct half of the super $\mathcal{N} = 1$ Virasoro algebra provided that the quadratic Casimir vanish, and the hamiltonian $H = L_{-1}$ is invertible. This condition amounts to the requirement that H has no ground state at $E = 0$ as is the case in (3.16). Equivalently, one can demand that supersymmetry is broken and there are no states which are annihilated by the supersymmetric generators. The full super Virasoro algebra is obtained when the special conformal generator $K = L_1$ is also invertible.

It would be interesting to find out what are the restrictions that higher supersymmetry [20] put on such constructions and if there are modifications that will account for central charges, since in the $\mathcal{N} = 1$ case the representation is not restricted only to quantum mechanics where one would expect these extensions to vanish.

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